United Kingdom
Mathematics Trust

Junior Mathematical Olympiad

Wednesday 14 June 2023

supported by Overleaf

England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Start each question on an official answer sheet on which there is a QR code.
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code.
6. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (*Your work will be scanned for marking.*)
7. Arrange your answer sheets in question order before they are collected. If you are not submitting work for a particular problem, please remove the associated answer sheet.
8. Your answers should be fully simplified and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
9. Only answers are required to the questions in Section A.
10. For questions in Section B, you should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Junior Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

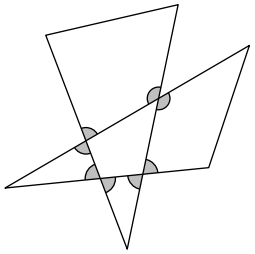
Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the integer nearest to $\frac{59}{13}$?

A2. What is the solution of the equation $24 \div (3 \div 2) = (24 \div 3) \div m$?

A3. Two triangles are drawn so that they overlap as shown.
What is the sum of the marked angles?

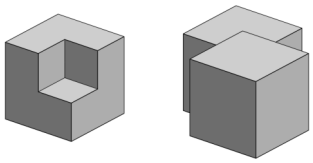


A4. What is the value of $\dfrac{(1^2+1)(2^2+1)(3^2+1)}{(2^2-1)(3^2-1)(4^2-1)}$? Give your answer in its simplest form.

A5. A number line starts at -55 and ends at 55. If we start at -55, what percentage of the way along is the number 5.5?

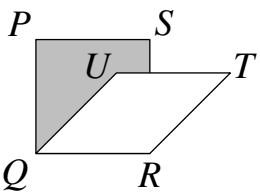
A6. Tea and a cake cost £4.50. Tea and an éclair cost £4. A cake and an éclair cost £6.50.
What is the cost of tea, a cake and an éclair?

A7. A $2 \times 2 \times 2$ cm cube has a $1 \times 1 \times 1$ cm cube removed from it to form the shape shown in the left-hand diagram. One of these shapes is inverted and put together with a second of the shapes on a flat surface, as shown in the right-hand diagram.
What is the surface area of the new shape?



A8. Alex chooses three from the six primes 2003, 2011, 2017, 2027, 2029 and 2039. The mean of his three primes is 2023.
What is the mean of the other three primes?

A9. The diagram shows the square $PQRS$, which has area 25 cm^2, and the rhombus $QRTU$, which has area 20 cm^2.
What is the area of the shaded region?



A10. What is the remainder when $\overbrace{23 \cdots\cdots 23}^{\text{twenty-three 23s}}$ is divided by 32?

Section B

Your solutions to Section B will have a major effect on your result.

Concentrate firstly on one or two Section B questions and then write out *full solutions* (not just brief 'answers'), including mathematical reasons as to why your method is correct.

You will have done well if you hand in full solutions to two or more Section B questions.

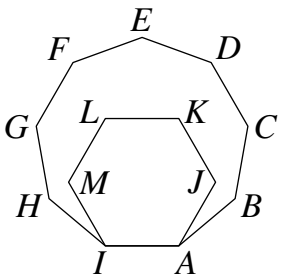
Do *not* hand in rough work.

B1. The sum of four fractions is less than 1. Three of these fractions are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{10}$. The fourth fraction is $\frac{1}{n}$, where n is a positive integer. What values could n take?

B2. Laura went for a training ride on her bike. She covered the first 10% of the total distance in 20% of the total time of the ride. What was the ratio of her average speed over the first 10% of the distance to her average speed over the remaining 90% of the distance?

B3. As Rachel travelled to school, she noticed that, at each bus stop, one passenger got off and x passengers got on, where $x \geq 2$. After five stops, the number of passengers on the bus was x times the number of passengers before the first stop. How many passengers were on the bus before the first stop?

B4. The regular nonagon $ABCDEFGHI$ shares two of its vertices with the regular hexagon $AJKLMI$.
Show that the points H, M and D lie on the same straight line.



B5. The eleven-digit number '$A123456789B$' is divisible by exactly eight of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. Find the values of A and B, explaining why they must have these values.

B6. The diagram shows five circles connected by five line segments.
Three colours are available to colour these circles.
In how many different ways is it possible to colour all five circles so that circles which are connected by a line segment are coloured differently?